PULSED POWER MEASUREMENT GUIDE

Prepared by Advance Power System

This guide is intended to educate those in the differences in power measurement involved with the Rosenberg cycle. The Rcycle is different than all classic methods of electric power use and some functions require understanding in the nuances of pulsed electric power.

The early electrical engineers, when confronted with this problem, acted as men always act. They first tried the various known devices on the new problem. The D'Arsonval meter would not indicate at all on most of the Alternating currents. The dynamometer meter essentially a fixed coil in series with a movable coil with pointer attached was found to indicate corresponding to a change in the alternating Current of the coils. The iron vane instrument fixed coil and pivoted strip of iron tilted with respect to the coil also changed in indication corresponding to a change in the alternating current of the coil. Finding an AC ammeter was thus a relatively simple matter. Determining more exactly what these meters indicated in terms of the actual current variation was not quite so easy, but it was still not very difficult.

In the first place, because the current to be measured is in both coils of a dynamometer meter, the deflection is proportional to the square of the current. Second, because the moving element of the meter, while relatively light in weight, still is very much too heavy to follow very rapid current variations, the meter indicates the average of the pull against its restoring spring. Thus the meter needle indicates the average of the square of the instantaneous current. But if direct current is observed with the same meter, the deflection is proportional to the square of the direct current also, and if the meter is to be used as an ammeter the scale will have to be redrawn so that it extracts the square root in order to indicate I and not I^2. Then, if this meter with the scale marked to measure direct current is used also to measure alternating current, the meter will in effect indicate the square root of the average of the squares of the instantaneous currents.

The mathematical expression for an average of a function is the definite integral of that function divided by the interval between the limits of integration area divided by the base. On the basis of the foregoing discussion of the properties of AC ammeters, they indicate in accordance with the expression.

TRUE RMS vs. ALTERNATIVES

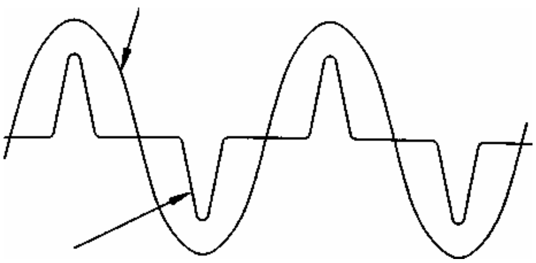
The waveform of voltage and current has long been assumed to be sinusoidal and fundamental. Therefore, most measuring equipment has been designed with these pure waves in mind. But as we have seen throughout this book, **many waveforms that we want to measure are not even close to sinusoidal. These unexpected waveforms can trick some meters into giving results that contain rather large errors**.

Two common metering techniques involve looking for the average or the peak of the voltage or current. The most inexpensive digital voltmeters use a form of averaging to produce readings. Since RMS = 1.11 times average, these meters look at the instantaneous voltage over a cycle, do a simple calculation, and display the result. Peak reading meters look for the highest instantaneous voltage and multiply that by .707.

Of course, the problem is obvious. Both of these types of meters anticipate the waveform to be that of a pure sine wave. The constants of .707 and 1.11 give consistent results with the actual RMS (root of the mean of the square) value only for a pure sine wave. True RMS digital meters sample the input at a high rate about 100 times the input frequency and convert these time slices into samples. A microprocessor squares each sample and sums the square along with those of each previous sample, then takes the square root of the sum. This method gives a precise calculation of true RMS regardless of the distortion of the wave shape being measured.

Sinusoidal Voltage True RMS = 200 Amps

Average Meter Reading = 100 Amps



Current Pulses

Figure 2. *This familiar pulse current signature must be measured with a "true RMS" meter. Cheap averaging meters could give reading of only half of the actual current!*

It should be noted that the most common meter among electricians today is an average reading RMS calibrated meter. If we look at the familiar pulse current signature of a switching power supply (Figure 2), a true RMS meter would give us an accurate reading. An averaging meter would read only about half as much, so if we measure neutral current with the wrong meter, we are liable to believe we don't have a problem at 12 amps when the true RMS current on that branch circuit neutral could be over 20 amps. The only proper meter to use to measure voltage and (especially) current is a true RMS meter. Anything else may be worse than taking no measurement at all.

DC COMPONENT

All waveforms have a direct current, or DC, component and an alternating current, or AC, component. Sometimes one component is zero. For example, in the output of a dry cell, the AC component is zero. In a 60-Hz household outlet, the DC component is zero because the average voltage from this source is zero. **In a complex waveform, the DC component is the average value of the voltage**. This average must be taken over a sufficient period of time. Some waveforms, such as the voltage at the collector of any amplifier circuit, have significant DC components.

AVERAGE ABSOLUTE PULSE AMPLITUDE

A pulse of voltage, current, or power is often irregularly shaped. The average absolute pulse amplitude is a measure of the pulse intensity. In determining the average absolute pulse amplitude, the beginning and ending times, t_0 and t_1, must be known. Then, the absolute value of the pulse polarity is taken by inverting the negative part or parts of the pulse, forming a positive mirror image. Next, the total area under the pulse curve is found. Finally, a rectangle is constructed, having the same beginning and ending times, t_0 and t_1 as the original pulse, and also the same area under the curve. The amplitude of this rectangular pulse is the average absolute pulse amplitude. The procedure for determining the average absolute pulse amplitude should not be confused with area redistribution (see AREA REDISTRIBUTION). While area redistribution involves the effective duration of an irregular pulse, **the average absolute-pulse amplitude is a measure of the effective strength, or intensity, of a pulse**.

AVERAGE VALUE

Any measurable quantity, such as voltage, current, power, temperature, or speed has an average value defined for a given period of time. **Average values are important in determining the effects of a rapidly fluctuating variable**.

Unless indicated otherwise, all sine-wave AC measurements are in RMS values. The capital letters V and I are used, corresponding to the symbols for dc values. As an example, V = 120 V for the AC power line voltage. The ratio of the RMS to average values is the form factor. For a sine wave, this ratio is 0.707/0.637 = 1.11.

To determine the average value of some variable, many instantaneous values (see INSTANTANEOUS EFFECT) are mathematically combined to **obtain the arithmetic mean** (see ARITHMETIC MEAN). The more instantaneous, or sampling, values used, the more accurate the determination of the average value. For a sine-wave half cycle (such as one of the pulses in A or B), having a duration of 10 milliseconds (0.01 second) at an AC frequency of 50 Hz, instantaneous readings might be taken at intervals of 1 millisecond, then 100 microseconds, 10 microseconds, 1 microsecond, and so on. This would yield first 10, then 100, then 1000, and finally 10,000 sampling values. The average value determined from many sampling values is more accurate than the average value determined from just a few sampling values. There is no limit to the number of sampling values that may be averaged in this way. The true average value however, is defined as the arithmetic mean of all instantaneous values in a given time interval. While the mathematical construction of this is rather complicated true average value always exists for a quantity evaluated over a specific period of time.

An equivalent method of evaluating the average value for a variable quantity calls for the construction of a rectangle, with a total area equal to the area under curve, for a certain time interval.

POWER

What is the power dissipated in a 4Ω light bulb connected to a 12V battery? What is the power dissipated a 2Ω light bulb connected to the same battery? Which bulb is brighter? Since the brightness of a bulb increases with the power, the 2Ω bulb is brighter than the 4Ω bulb. That is, power is changed step-for-step with voltage. The changing resistance maintains a constant current despite changes of voltage. At any point on the graph, voltage divided by resistance is 1 ampere. Had the current been allowed to vary, power would have changed at a curved, or square rate, instead of linearly as on the graph.

Up to this point, four of the most important basic electrical quantities have been discussed. These are E, I, R, and P. It is of fundamental importance that you thoroughly understand the interrelation of these quantities. You should understand how any one of these quantities either controls or is controlled by the others in an electrical circuit. These relations are further explained in the treatment that follows. You should compare each statement carefully with its associated formula.

This formula states that P is the product of E multiplied by I, regardless of their individual values. If either E or I vary, P varies. If both E and I vary, P varies at a square rate.

This formula states that if R is held constant, and I is varied, P varies as the square of I, because I appears as a squared quantity (I^2) in the formula. Also, if I is held constant and R is varied, P varies directly and proportionally to R, because R is a multiplier in the formula. Power, as related to E and R This formula states that if R is held constant as E is varied, P varies as the square of E, because E appears as a squared quantity (E) in the formula. Also, if E is held constant and R is varied, P varies inversely but proportionally to R, because R is a divisor in the formula.

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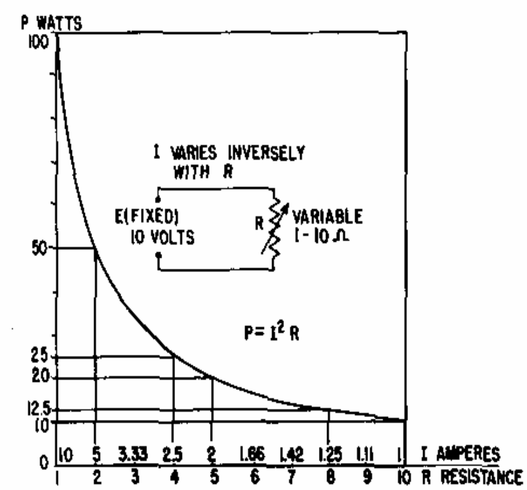
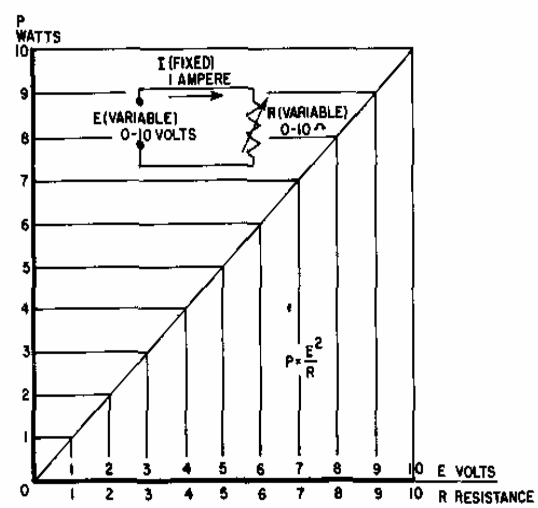




WHAT IS THE POWER OF AN INCANDESCENT LAMP?

Shows a potential difference of 120 V placed across a circuit that has a lamp with resistance R1 = 144 Ω connected in series to a variable resistor R_2. Changing the magnitude of R2 controls the brightness of the lamp. Find the power dissipations in the lamp (a) when R_2 is zero and (b) when R_2 = 144Ω. (c) What must R_2 be for the power dissipation in the lamp to be 50 W?

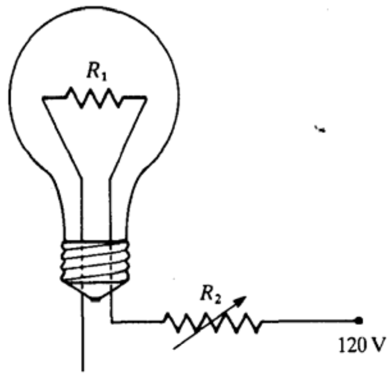


Fig. 27-33

$$I = \frac{V}{R_1} = \frac{120\ \text{V}}{144\ \Omega} = 0.833\ \text{A} \qquad P_1 = R_1 I^2 = (144\ \Omega)(0.833\ \text{A})^2 = \underline{100\ \text{W}}$$

$$I = \frac{V}{R_1 + R_2} = \frac{120\ \text{V}}{288\ \Omega} = 0.4165\ \text{A} \qquad P_1 = R_1 I^2 = (144\ \Omega)(0.4165\ \text{A})^2 = \underline{25\ \text{W}}$$

$$I^2 = \frac{P_1}{R_1} = \frac{50\ \text{W}}{144\ \Omega} = 0.347\ \text{A}^2; \qquad I = 0.589\ \text{A} \qquad R = R_1 + R_2 = \frac{V}{I} = \frac{120\ \text{V}}{0.589\ \text{A}} = 203.7\ \Omega$$

$$R_2 = R - R_1 = \underline{59.7\ \Omega}$$

WHAT IS MEANT BY THE ROOT-MEAN-SQUARE (RMS) VALUE OF AN ALTERNATING CURRENT?
When an alternating emf, one fluctuating with simple harmonic motion: $\delta = \delta_p \cos(t + \delta)$ is the energy source in a circuit containing only resistors, the current fluctuates in the same way: $I = I_P \cos(\omega t + \delta)$. Here δ_p is the peak emf, is the instantaneous emf, ω is the angular frequency, and δ is the constant phase angle. l_p and I are peak and instantaneous current. δ_{ms} is the square root of the time average of δ^2 over a complete cycle. It can be shown that the time average of $\cos^2(\omega t + \delta)$ over a complete period is equal to 1/2.

Then (δ^2) avg = $1/2\ \delta^2_p$ and $\delta_{RMS} = \delta_p/\sqrt{2}$. Similarly $I_{ms} = I_p/\sqrt{2}$. For a circuit with total equivalent resistance R, we have $\delta = IR$ or, canceling the cosine term on both sides, $\delta_p = I_P R$. Then $\delta_{ms} = I_{ms}R$. The average power dissipated is $(I^2 R)_{avg} = (I^2)_{avg} R = ½\ I^2_P R = I^2_{ms} R$. The average power supplied by the emf is $P = P_{avg} = (\delta\ I)_{avg} = \delta_p l_p(\cos^2(\omega t + \delta))_{avg} = 1/2\ \delta_p I_P = \delta_{rms} I_{rms}$. Similar reasoning shows that for any individual resistance with alternating current I, V = IR => $V_{rms} = I_{rms} R$, where V is the alternating voltage across the resistance.

EFFECTIVE OR RMS VALUE
As the use of alternating current gained popularity, it became increasingly apparent that some common basis was needed on which AC and DC could be compared. A 100-watt light bulb, for example, should work just as well on 120 volts ac as it does on 120 volts DC It can be seen, however, that **a sine wave of voltage having a peak value of 120 volts would not supply the lamp with as much power as a steady value of 120 volts DC**.

Since the power dissipated by the lamp is a result of current flow through the lamp, the problem resolves to one of finding a MEAN alternating current ampere, which is equivalent to a steady ampere of direct current. A circuit in which the peak alternating current through the 10-ohm resistor is 1.414 amperes. Since the current through the resistor is changing continuously the power dissipated by the resistor will also vary. It will be minimum when the current is zero. The variations in power throughout the cycle can best be analyzed by plotting a curve showing the instantaneous power at each point in the cycle. In the procedure to follow, the instantaneous current, the square of the instantaneous current, and the instantaneous power will be calculated in 10° steps for the first quarter of the cycle.

ROOT MEAN SQUARE

The current, power, or voltage in an alternating current AC signal can be determined in various ways. The most common method of expressing the effective value of an AC waveform is the root-mean-square (RMS) method. The RMS current, power, or voltage is an expression of the effective value of a signal.

The RMS current, power, or voltage is determined with the following procedure. First, the amplitude is squared so that the negative and positive halves of a waveform are made identical. Then the value is averaged over time. Finally, the square root of the average square value is determined. Mathematically, the RMS value is an expression of the DC effective magnitude of an AC or pulsating direct current waveform. For a sine wave, the RMS value is 0.707 times the peak value, or 0.354 times the peak-to-peak value. **For a square wave, the RMS value is the same as the peak value, or half the peak-to-peak value. For other waveforms, the ratio varies**.

ROOT-MEAN-SQUARE AMPLITUDE

Often it is necessary to express the effective amplitude of an ac wave. This is the voltage, current, or power that a dc source would produce to have the same general effect in a real circuit or system. When you say a wall outlet has 117 V, you mean 117 effective volts. The most common figure for effective ac levels is called the root-mean-square, or rms, value.

The expression root mean square means that the waveform is mathematically "operated on" by taking the square root of the mean of the square of all its instantaneous values. The rms amplitude is not the same thing as the average amplitude. For a perfect sine wave, the rms value is equal to 0.707 times the peak value, or 0.354 times the pk-pk value. Conversely, the peak value is 1.414 times the rms value, and the pk-pk value is 2.828 times the rms value. The rms figures often are quoted for perfect sine-wave sources of voltage, such as the utility voltage or the effective voltage of a radio signal.

For a perfect square wave, the rms value is the same as the peak value, and the pk-pk value is twice the rms value and twice the peak value. For sawtooth and irregular waves, the relationship between the rms value and the peak value depends on the exact shape of the wave. The rms value is never more than the peak value for any wave shape.

THE RMS VALUE OF A SINE WAVE

Instantaneous values of alternating current and voltage are not satisfactory for most measurement purposes since the pointers of meters cannot follow the rapid changes in polarity. Nor are they suitable for calculation purposes because by the time we complete the computation, the instant to which the calculation applied has long since passed. Full-cycle average values of sine waves are zero, and half-cycle average values are only useful for such applications as rectification where we are concerned primarily with a half-cycle of a sine wave.

These problems do not arise in DC circuits since the voltage, current, and power in a resistive circuit remain constant for a considerable period of time. **As far as the end result of converting**

electric energy into light is concerned, it makes no difference whether the lamp is lighted by a direct current or an alternating current flowing through it. Perhaps, then, we can establish equivalent steady-state values for alternating current and voltage which will allow us to use the same interrelationships among voltage, current, resistance, power, work, etc. that we use in DC circuits. We can determine this equivalent DC or effective value of an alternating current experimentally by finding out what direct current must flow through a given resistance to produce in a certain time interval the same amount of heat energy as when the resistor is connected to a source of alternating voltage.

To determine this equivalent DC value of an alternating current, we can conduct the same experiment algebraically with pencil and paper. The basis for comparison in the experiment is the same amount of heat energy in a certain time interval. We can translate this into average work per unit time or simply average power. To determine average power in a resistive AC circuit, $P = I^2 R$.

Using the averaging technique we compute the square of the instantaneous current at small time intervals over a full cycle and then calculate the average or mean value of all these I^2 values. Hence, Pav = (full-cycle average of I^2) x R.

In a DC circuit, $P = I^2R$. Since the effective values of an alternating current are to represent DC equivalent values, we can use the same letter symbols for both, that is, uppercase italic letters without subscripts. Therefore, in an AC circuit, $P= I^2$ where P is average power and I is the DC equivalent value of the alternating current. We can define the DC equivalent value of an alternating current in terms of the process we go through to determine its value. We find the square root of the mean value of the squares of the instantaneous current over a full cycle. Root-mean-square (usually abbreviated to RMS), effective and equivalent DC is synonymous terms.

For a sine wave of alternating current, we note that the instantaneous power swings alternately and uniformly between zero and peak power. It appears quite reasonable to state, therefore, that the average power in a resistor through which a sine-wave alternating current is flowing is one-half the peak power.

We can also determine this relationship from the general Equation for instantaneous power: $P = P_m \sin^2 \omega t$. If we average a sine or cosine function over a complete cycle, the average must be zero, since for every positive value during the first half-cycle, there is an equivalent negative value during the second half-cycle. Hence, when averaged over a complete cycle, becomes zero and Equation becomes $P = 0.5\ P_{m.}$

The RMS value of a sine wave voltage should be such that the average power is the product of the RMS voltage across and RMS current through the resistance of the circuit, just as in the equivalent DC circuit. Therefore, P = VI. But when a sine wave current flows through a resistor, $P = 0.5P_{m.}$

PEAK AMPLITUDE

One of the most frequently measured characteristics of a sine wave is its amplitude. Unlike DC measurement, the amount of alternating current or voltage present in a circuit can be measured in various ways. In one method of measurement, the maximum amplitude of either the positive or the negative alternation is measured. The value of current or voltage obtained is called the PEAK

VOLTAGE or the PEAK CURRENT. To measure the peak value of current or voltage, an oscilloscope or a special meter (peak reading meter) must be used.

PEAK-TO-PEAK AMPLITUDE

A second method of indicating the amplitude of a sine wave consists of determining the total voltage or current between the positive and negative peaks. This value of current or voltage is called the PEAK-TO-PEAK VALUE. Since both alternations of a pure sine wave are identical, the peak-to-peak value is twice the peak value. Peak-to-Peak voltage is usually measured with an oscilloscope, although some voltmeters have a special scale calibrated in peak-to-peak volts.

THE PEAK VALUE OF A SINE WAVE

As we discovered in determining the nature of an alternating voltage, the greatest value that the induced voltage can attain occurs when ø = 90°. At this point, the rate of cutting of magnetic lines of force is maximum. When ø = 270°, the same maximum value is attained but the polarity of the induced voltage is reversed. The numerical value that the alternating voltage attains at these two angles is termed the peak value of the AC waveform. The letter symbol for the peak value of an alternating source voltage is E_m ($_m$ for maximum). This peak value is a numerical quantity in volts and does not have any polarity, since the same peak value in volts occurs at both 90° and 270°. The peak value is also independent of time, since, no matter where the rotating conductor may be at a certain instant, the peak value will appear as it passes the 90° and 270° points in its rotation. Therefore, the peak value is a constant for a given generator and does not have the sine-wave shape of the voltage induced from instant to instant.

In the simple AC generator, the peak voltage will depend on just what the maximum rate of cutting of magnetic lines of force by the rotating loop happens to be. This rate, in turn, depends on (1) the total number of lines of force in the magnetic field, (2) the angular velocity of the loop, and (3) the number of turns of wire in the loop. From Faraday's law, when a conductor cuts across magnetic lines of force at the rate of one Weber per second, an emf of one volt is induced in that conductor.

THE INSTANTANEOUS VALUE OF A SINE WAVE

Any value that is continually changing and thus is dependent on the exact instant in time for its numerical value is called an instantaneous value. Therefore, in representing the instantaneous value of the voltage induced in the rotating loop of the simple AC generator, we must use the lower case letter symbol e.

In the simple AC generator during the first 180° of rotation of the loop, the voltage rose to a peak value in one direction and dropped back to zero; then, during the second 180° of rotation, it rose to a similar peak value but in the opposite direction and again dropped back to zero. Therefore, it took one complete revolution for the voltage to vary through the whole possible range of values. As the loop rotates past 360°, the same series of events recurs. We define this complete excursion of the instantaneous value of the induced voltage as one cycle of a sine wave. The time it takes for the instantaneous value to complete one cycle is called the period of the sine wave. The number of cycles completed in one second is called the frequency of the sine wave.

PHASE ANGLE

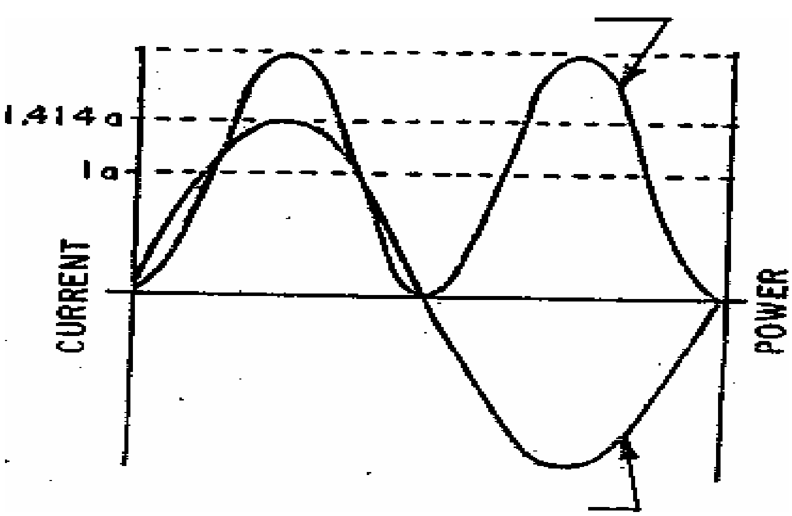
The voltage induced into the rotating conductor at any point is directly proportional to the sine of the angle through which the loop has rotated from the reference axis. We call this angle the phase angle. Determined by geometrical construction the relative magnitude of an induced voltage for an angle ø, which is less than 90°. In its final form, the procedure amounted to drawing a perpendicular from the tip of the Phasor (point X) to the horizontal reference axis (point V). As long as the length of the Phasor RV OX remains constant, the magnitude of the induced voltage is directly proportional to the length of XV. Exactly the same geometrical construction applies to angular distances of more than 90° from the reference axis. The Phasor OX has rotated through an angle of 120°. When we draw a perpendicular from point X to the reference axis, we form a right-angled triangle in which the side XV is opposite an angle that is equal to 180° or 60°.

Notice that at 0° the instantaneous current (I) is zero causing the power dissipated by the resistor to be zero. At 10° the instantaneous current is 0.245 amperes, the current squared is 0.060 and the power is 0.60 watt. At 90° the current has reached its maximum value of 1.414 amperes, the square of the current is 2.000 and the power dissipated is 20.00 watts.

During the part of the sine wave of current from 90 °to 180° the same values could be used as before but in a reverse order. Thus, at 100° the values of current and power would be identical to those at 80 °. Using the values of I and P a graph can be constructed showing the way in which power varies throughout the cycle. In graph a sine wave of current is plotted first, using the instantaneous values. Next the curve representing I^2 and power is constructed. Notice that the power curve has twice the frequency of the current curve, and that ALL POWER IS POSITIVE. This is due to the fact that heat is dissipated regardless of which way the current flows through the resistor. Since the alternations of the power curve are identical, the MEAN or AVERAGE POWER is the value HALF WAY between the maximum and minimum values of power. Thus, the average power dissipated by the 10-ohm resistor is 10 watts, one half the peak power. Since the curve representing power also represents current squared (I^2) the average or mean of the curve also lies halfway between the maximum and minimum values of I^2. As power is proportional to I^2, a DC current having a value equal to the square root of the mean of the I^2 values would produce the same average power as the original sine wave of current. This mean current is called the ROOT MEAN SQUARE (RMS).

CONDUCTION DELAY ANGLE

The time from AC wave zero crossing, until the Thyristor fires and conducts. A portion of the source sine wave will be conducted into the load. Past 90 degrees the voltage of the conduction rapidly decreases to zero.



ROOT MEAN SQUARE (RMS) current

One RMS ampere of alternating current is as effective in producing heat as one steady ampere of direct current. For this reason a RMS ampere is also called an EFFECTIVE ampere. The peak current of 1.414 amperes produces the same amount of average power as 1 ampere of effective (RMS) current. ANYTIME AN ALTERNATING VOLTAGE OR CURRENT IS STATED WITHOUT ANY QUALIFICATIONS, IT IS ASSUMED TO BE AN EFFECTIVE VALUE. Since effective values of AC are the ones generally used, most meters are calibrated to indicate effective values of voltage and current.

In many instances it is necessary to convert from effective to peak or vice-versa. The peak value of a sine wave is 1.414 times the effective value and therefore:

E_m = Ex 1.414 E_m = maximum or peak voltage

E = effective Or RMS voltage

I_m = I x 1.414

I_m = maximum or peak current I = effective or RMS current.

Upon occasion it is necessary to convert a peak value of current or voltage to an effective value. The conversion factor may be derived as follows:

E_m = E x 1.414

Multiplying both sides of the equation by 1/1.414

$$E_m \times \frac{1}{1.414} = E \times 1.414 \times \frac{1}{1.414}$$

$$E_m \times \frac{1}{1.414} = E$$

Dividing 1 by 1.414

E = E x 0.707 m

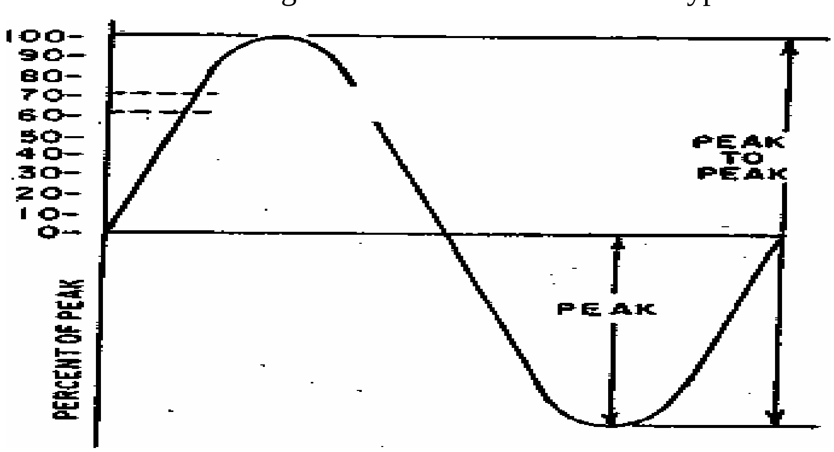
Where E = the effective voltage E = the maximum or peak voltage m

Similarly for current

I = I_{mp} x 0.707 where I = the effective current I_{ra}= the maximum or peak current.

AVERAGE VALUE

The average value of a complete cycle of a sine wave is zero, since the positive alternation is identical to the negative alternation. In certain types of circuits however, it is necessary to



compute the average value of one alternation. This could be accomplished by adding together, a series of instantaneous values of the wave between 0° and 180°, and then dividing the sum by the number of instantaneous values used. Such a computation would show one alternation of a sine wave to have an average value equal to 0.637 of the peak value. In terms of an equation: E x 0.637.

THE AVERAGE VALUE OF A PERIODIC WAVE

By definition, an alternating current (or voltage) is one in which the average of the instantaneous values over a period of time is zero. Just by examining waveforms it is fairly obvious that the sine wave, square wave, and saw tooth wave are alternating currents (or voltages), since the instantaneous values are symmetrical about the horizontal axis of the graph. The sine-squared wave, pulse wave, and half-wave rectified wave do not qualify as alternating currents, since their instantaneous values are always of the same polarity.

We are quite familiar with the technique for averaging examination marks or determining batting averages in baseball. The same technique applies to determining the average value of any periodic wave. For complex periodic waves we can determine the average of the instantaneous values graphically by examining the area under a graph. But for the sine wave, we can obtain a more accurate average value by writing down the sine of 5°, 10°, 15°, and so on, for a complete cycle. To find the average value of any column of numbers, we add the column of figures and divide by the number of figures in the column. After going to all this trouble, the average value of a sine wave for a complete period is zero, since, for every positive value of instantaneous voltage or current during the one half-cycle, there is a similar negative value during the next half-cycle.

However, if we average one-half cycle only, we obtain a significant numerical constant. From the column of figures prepared as above from a sine table or calculator, we find that the average of sin ωt over one half-cycle is 22.9 / 36 = 0.636. If we average the half-cycle of a sine wave by integration (area under the curve method), we arrive at a slightly more accurate result, expressing average values as 2/π or 0.637.

Since $I = /_m \sin \omega t$.: $Iav = I_m \times 0.637$

Therefore, the half-cycle, average value of a sine wave is

$_{Iav} = 0.637/_m = 2/\pi\, _{Im}$

or $E_{av} = 0.637Em = 2/\pi\ Em$

As we discovered the sine-squared wave will have a full-cycle average value of one-half the peak value of the sine-squared wave.

Pulse waveform will have an average value that depends on the ratio of the pulse duration to the time interval between pulses. The half-wave rectified sine wave will have a full-cycle average value which, in turn, is the average of $0.637E_m$ for one half-cycle and zero for the next half-cycle, or $0.3183E_m$.

INSTANTANEOUS CURRENT IN A RESISTOR

Now that we are acquainted with the sinusoidal nature of the instantaneous voltage generated by a basic AC voltage source, we can determine the nature of the current that will flow when we connect resistance to the generator terminals in the simple AC circuit. At a particular instant in time, the instantaneous voltage developed by the generator has a certain magnitude and a certain polarity. If we consider only this one particular instant, there is no difference between the simple AC circuit and the equivalent DC circuit. The real difference between them is that, at the next instant in time, the magnitude of the instantaneous alternating voltage changes, but the applied voltage in the DC circuit continues on at a fixed magnitude. If we consider any particular instant in time. Therefore, we can state that the instantaneous current through the resistance where I is the instantaneous current through the resistance, e is the instantaneous voltage applied to the resistance, and R is the resistance of the circuit.

Since resistance is determined by such physical factors as type of material, length, and cross section, it is a constant for a given circuit at a given temperature indicated by an uppercase R in Equation. Because R is a constant, whatever the variation in instantaneous voltage, the instantaneous current must stay right in step in order to make Ohm's law hold true. If the instantaneous voltage is a sine wave, the instantaneous current must also be a sine wave. The instantaneous current must reach its peak value at the same instant that the instantaneous voltage becomes maximum. It will become zero at the same instant that the instantaneous voltage becomes zero, and it will reverse its direction at the same instant that the instantaneous voltage across the resistance reverses its polarity.

INSTANTANEOUS POWER IN A RESISTOR

If we still think in terms of instantaneous values, Therefore, where p is the instantaneous power in a resistor in watts, v is the instantaneous voltage drop across the resistance in volts, I is the instantaneous current through the resistance in amperes, and R is the resistance of the circuit in ohms.

As R is a constant for a given circuit, and since the instantaneous voltage and current in the basic circuit are both sine waves, the instantaneous power must be a sine-squared wave. To determine the nature of a sine-squared wave, we can plot a graph of instantaneous power in the same manner that we plotted the sine curve. If we calculate the squares of the sine plots on a linear graph, the instantaneous power in a resistor takes on the shape. Note that the instantaneous power in a resistor is always positive since squaring a negative quantity results in a positive quantity. We can interpret positive power as electric energy being converted into some other form of energy (heat in the case of a resistor). On this basis, negative power would represent some other form of energy being converted into electric energy. A resistor is not capable of doing this; therefore, the waveform of instantaneous power in a resistor has no negative component.

The instantaneous power in the basic AC system is pulsating in nature, swinging from zero to maximum and back twice each cycle. If we consider an electric lamp as the resistance as far as heating the filament is concerned, it does not matter which way the current flows through it. There will be one pulse of energy converted into heat and light on the one half-cycle of current and another similar pulse on the second half-cycle. If the lamp is operated from a 60-hertz source, it will increase in brilliance 120 times a second. Therefore, the instantaneous power pulsations are at twice the frequency of the voltage and current. This pulsating characteristic of instantaneous power is responsible for the flicker in 25-hertz lighting as used in some localities and for the vibration of small ac motors.

Joining plots gives us the curve representing power input to a resistor. Since the voltage drop across a resistor must be exactly in phase with the current through it, the instantaneous voltage and current reach their peak values simultaneously. Whenever I is a positive quantity, v is also a positive quantity; and whenever I is a negative quantity, v is also a negative quantity. Since the product of two negative quantities is a positive quantity, the instantaneous power graph for a resistor is always positive. This indicates that a resistor must always convert electric energy into heat, never vice versa.

We also noted that the instantaneous power in a resistor is pulsating in nature, fluctuating sinusoidally between zero and a peak value of P_m = Vm Im twice during each complete cycle of voltage and current. Because of this smooth sinusoidal variation between zero and Pm, we concluded that, as far as doing work is concerned, we can consider the power in a resistive AC circuit as having an average, or DC equivalent value equal to 0.5 Pm.

We verified this conclusion by substituting the sine-wave form for the instantaneous voltage and current, giving: P = Vm lm sin 2wt = Pm sin 2wt. From trigonometric half-angle relationships, this became P = 0.5 Pm (l - cos 2wt). Since the average of a cosine wave over a complete cycle is zero, the average power P= 0.5 Pm.

Average value of power in a resistor establishes the RMS value of a sine wave of voltage and current as 0.707 of its peak value. Therefore, it follows that the average power input to the resistance of an AC circuit is simply the product of the RMS value of the voltage drop across and the RMS value of the current through the resistance, and P = $V_R I_{R.}$

Because a resistor can convert energy only from an electric form into heat or light but never vice versa, the average power in watts determined by Equation used to be called the true power input to the resistance of an AC circuit. More recent terminology refers to the average power input to the resistance of an AC circuit as real power or active power. We shall appreciate the choice of such designations when we consider power in an AC circuit containing both resistance and reactance. Note that the letter symbol for average power in an AC circuit is the same as that for power in a DC circuit.

The product of the RMS voltage across and the current in the resistance of an AC circuit is called the active power of the circuit. The letter symbol for active power is P. Since R = V_R /I_R, just as in DC circuits.

CONSERVATION OF ENERGY

The air, as we wave a hand through it, seems to be perfectly continuous. Yet on a fine enough scale, air is not continuous at all but comes in "lumps," that is, in particles of specific masses mainly oxygen and nitrogen molecules. We say that mass is quantized. We would all agree that a swinging pendulum can have any reasonable energy that we choose to give it. Things are different in the atomic world, however. An atom can exist in only certain characteristic states called quantum states each associated with a discrete energy E.

Allowed energy values (or energy levels) for an isolated sodium atom, each value corresponding to a different quantum state. The lowest energy, labeled E_0 and usually assigned the arbitrary value of zero, is the ground state of the sodium atom. It is the state in which an isolated sodium atom usually exists, just as a marble in a bowl usually rests at the bottom of die bowl. To attain a higher excited state, the sodium atom must absorb energy from some external source, perhaps by colliding with electrons in a sodium vapor. When it returns to its ground state, it must decrease its energy, perhaps by radiating light.

Actually, the energy of anything consisting of atoms including a swinging pendulum is quantized. However, for large objects the allowed energy values are so close together that they cannot be distinguished and appear to be continuous. For such objects, we can ignore quantization totally because we cannot possibly detect it (or its effects).

POWER FACTOR

Energy consumption is related to power. Thus the electric utility bases its rates on watts or kilowatts. But we saw from the preceding section that not all current is converted to power. We saw that kilowatts are not necessarily equal to kilovolt-amperes. These two units, however, are related by a ratio called power factor (PF). Power factor is an indication of the portion of volt-amperes that is actually power. To find power factor, the following formula can be used:

$$PF = \frac{\text{Power}}{\text{volt-amperes}}$$

P / Papp

The product of voltage and current is called apparent power. Wattage represents true power. The product of the voltage across a reactance and the current through the reactance is called reactive power. Power factor is therefore true power divided by apparent power. It is commonly expressed as a percentage. When true power is equal to apparent power such as in a circuit having only resistance or one in which the reactances exactly cancel one another the power factor is equal to 1.00 or 100 percent. (To obtain percentage from the power factor ratio formula, multiply the answer by 100.) Reactances consume no true power, and so the power factor of capacitance and inductance is 0 percent.

WHAT IS POWER?

Power is the time rate of doing work: $P = \frac{dW}{dt} = F \bullet v = Fv \cos ø$.

Where F and v are the instantaneous force and velocity, respectively, and ø is the angle between F and v. If the power does not vary with time, P = W/t. The units for power are joules per second. This combination of units is termed the watt. Therefore, $1\ W = 1\ J/s = 1\ kg\ X\ m^2/s^3$. Another unit is horsepower; 1 hp = 550 ft • Ib/s = 746 W.

MEASURING TRUE AND APPARENT LINE POWER

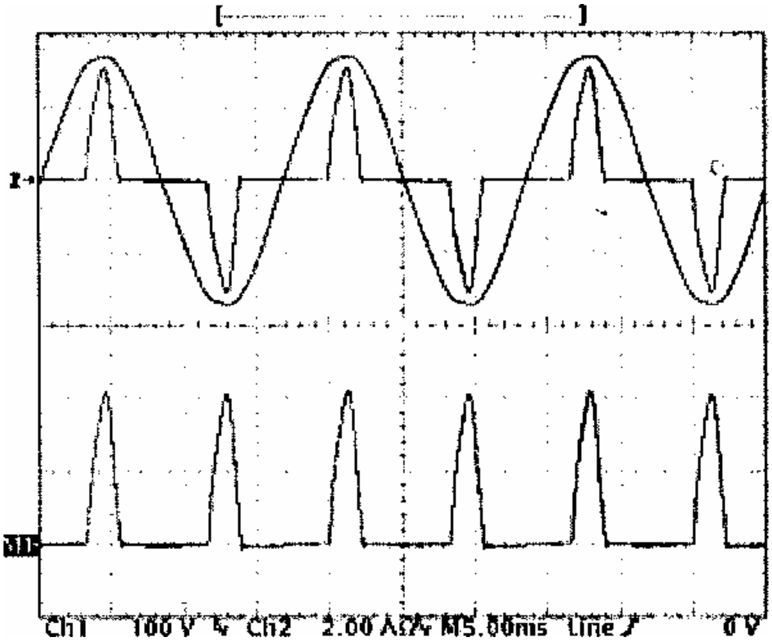
Problem: Erroneous results in AC power measurements due to inadequate sample rate and record length. Quantifying AC line-power components is a basic requirement in every off-line power converter application. The digital scope is an ideal tool for this task, but some subtle traps can turn this straightforward measurement into a tedious exercise. The first step is to acquire the voltage and current waveforms. A standard 10X passive probe can safely sense the line voltage. The scope can readily sense the line current. The upper waveforms show the distorted current and sinusoidal voltage waveforms. The scope calculates the individual RMS values of 121 Volts and 1.11 Amps. The apparent power is 134 VA or the product of the individual RMS values. True power is the mean value of the instantaneous product of voltage and current. The TDS multiplies the voltage and current waveforms to create the instantaneous power waveform. The TDS then calculates the mean value of the entire power waveform, which yields a true power of 88 W. The ratio between the true and apparent power values yields the power factor of 0.66.

Apparent Power (PA) = 20.8 V 1.108 A -133.8 W*
True Power = (Pt) = 88.0 W
Power Factor (pf) =88.0 W = 0.66 133.8 W

Contrast this with AC current transformer (CT) clamps. Many CT devices are not suitable for sensing the complex waveshapes at switching converter inputs. While rated for 50 or 60 Hz operation, their low-frequency roll-off can induce phase shift with respect to the voltage measurement. This phase shift will cause errors in the true power measurement since the two waveforms are multiplied in time. It's important to note that CT devices will very accurately sense the current's fundamental frequency component, but they're not necessarily designed to sense the higher frequency harmonics (up to several kHz) found in contemporary power waveforms.

As with any measurement of a periodic signal, a rule is to record and measure over complete events. In the case of AC waveforms, this means selecting a measurement interval that includes an integral number of line cycles. With some planning, you can set your scope to capture integral numbers of 60 Hz (or 50 Hz) cycles. The waveforms are exactly 50 milliseconds long or three complete line cycles. This results from a sampling rate of 50 kS/s (i.e., 20 ųs between samples) and a record length of 2500 samples. For example, a sample rate of 20 kS/s and a record length of 1024 samples result in 3.07 line cycles (instead of 3.00 cycles), which translates to a calculation error of greater than 2%.

In some cases, you can obtain acceptable results by using a digital scope's cycle-based measurement functions. The scope scans the selected waveform for a complete cycle of data and performs the measurement only on a complete cycle of the waveform. This technique works well with simple sinusoidal signals such as measuring the RMS value of the line voltage, but **it can lead to erratic results with complex current and true power waveforms**. In addition, a single cycle measurement of the 120 Hz instantaneous power waveform only represents half of a 60 Hz line cycle.



The upper waveforms are the voltage and current scaled at 100 Vper division and 2 A per division. The scope multiplies the voltage and current to create the instantaneous power waveform (lower).

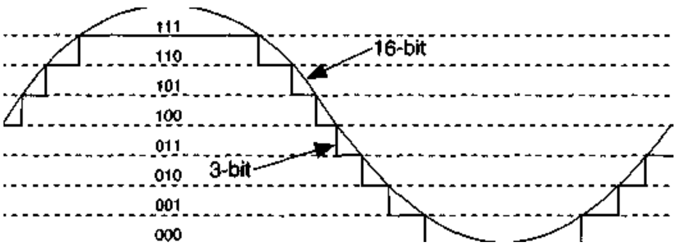
Of course there are applications where you want to directly control the measurement interval. You may want to measure the difference between the power delivered on the two halves of the line cycle or test your converter's power consumption at 47 Hz using a programmable AC source. In these cases, you would use gated measurement capability to directly set the time interval for a calculation.

This assumes your AC neutral and ground are at the same potential. If your building's ground wiring has a "power-quality" problem, or if you need to actually measure the line-to-neutral voltage, use the differential probe. Do not connect the ground clip of your standard 10 X passive probe to the AC neutral. Under no circumstances should you "float" your scope by cutting or defeating the ground connection.

SAMPLING DATA

You have an analog signal, so you must convert the signal with a DC measurement system, which converts your signal into information you and the computer can understand. Some of the issues you must resolve before choosing a measurement system are your ADC bit resolution, device range, and signal range.

The number of bits used to represent an analog signal determines the resolution of the ADC. You can compare the resolution on a DAQ device to the marks on a ruler. The more marks you have, the more precise your measurements. Similarly, the higher the resolution, the higher the number of divisions into which your system can break down the ADC range, and therefore, the smaller the detectable change. A 3-bit ADC divides the range into 2^3 or 8 divisions. A binary or digital code between 000 and 111 represents each division. The ADC translates each measurement of the analog signal to one of the digital divisions. A sine wave digital image as obtained by a 3-bit ADC. Clearly, the digital signal does not represent the original signal adequately, because the converter has too few digital divisions to represent the varying voltages of the analog signal. By increasing the resolution to 16 bits, however, the ADC's number of divisions increases from 8 to 65,536 (2^{16}). The ADC can now obtain an extremely accurate representation of the analog signal.



Range refers to the minimum and maximum analog signal levels that the ADC can digitize. Many DAQ devices feature selectable ranges, so you can match the ADC range to that of the signal to take best advantage of the available resolution. For example, in Figure 5-5, the 3-bit ADC, as shown in the left chart, has eight digital divisions in the range from 0 to 10 volts. If you select a range of-10.00 to 10.00 volts, as shown in the right chart, the same ADC now separates a 20-volt range into eight divisions. The smallest detectable voltage increases from l. 25 to 2.50 volts, and you now have a much less accurate representation of the signal.

THE RADIAN

In dealing with the rotating conductor we have been talking about angular distance and angular velocity. Although the Babylonian 360° system of angular measurement is very convenient for purposes of geometry and trigonometry, it is not suited to showing the relationship between the linear velocity of the conductor as it moves around the circumference of a circle and its angular velocity. The linear velocity of the conductor governs the actual rate of cutting across magnetic lines of force. Therefore, for electrical purposes, it would be very useful to have a unit of angular distance, which is related to the distance traveled by the free end of a rotating Phasor. The radian is just such a unit. By definition, one radian is the angular distance through which the Phasor travels when its free end travels through a linear distance equal to the length (OX) of the Phasor. It is standard practice in electrical engineering to express the angular distance traveled by a rotating Phasor in radians, and its angular velocity in radians per second. Since the rotating Phasor is the radius of a circle, and the circumference of a circle = 2irr, there are 2ir radians in one complete cycle. Hence, in marking the x-axis of a time graph in radians.

We must also convert from angular distance in degrees to angular distance in radians. Most engineering calculators will allow us to solve trigonometric functions in radians without having to convert back to degrees. If we do have to convert radians into degrees for some reason, **the conversion factor becomes 2 π rad = 360° and 1 rad = 57.3°**.

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